

TEACHING better MATHEMATICS: Collaborative inquiry into mathematics learning and teaching (Barbara Jaworski, 21.11.2006)

In March 2006 a Consortium of 5 Colleges submitted a proposal (with title as above) to the RCN Praksisrettet FoU programme focused on sections 1 and 3 of the PFoU programme, <http://www.forskningsradet.no/praksisfou/>. The proposal was selected for funding, but the amount of funding offered was substantially less than that requested. A revised proposal had to then be submitted to fit the offered funding. This revised proposal was accepted. Thus the TBM project has started officially in autumn 2006.

This document sets out the main principles behind the projects, consortium-wide, and then offers a description of each of the projects as they are conceptualized in the early stages of the programme.

Statement of the main aims and goals of the proposed consortium

To develop and improve mathematics education in Norway with special reference to the improvement of learning experiences for pupils in mathematics classrooms and to the education of new mathematics teachers.

Background to the proposal

Mathematics education in Norway

Mathematical achievement in Norway is not as strong as educators or society in general would like to see. This is reflected in both international and national studies such as TIMSS, PISA, KIM and the L97 evaluation (Lie, Kjaernsli & Brekke; Brekke, 1995; Alseth, Breiteig, & Brekke, 2003). Students in schools are achieving below international averages. National studies show many areas of mathematics to be problematic for pupils (Brekke, 1995). An evaluation of the new curriculum introduced in 1997 (L97: The Royal Ministry of Education, Research and Church Affairs, 1999) suggests, for some areas of mathematics, that pupils are doing less well than in a review of the earlier curriculum (Alseth et al, 2003).

Since Norwegian schools are organised according to year or grade groups, in grades 1 to 10 (grunnskole: ages 6-15), within a year or grade group there may be several mixed-ability classes taught by members of the year group teacher team. Planning within these groups is designed overtly for social inclusion and diversity of experience. Thus teachers need to differentiate across a wide range of experience and achievement in mathematics as well as in other subjects. It is hard for a teacher to know every subject in depth, so it means, for many pupils in many subjects, that subject specialisation is left until the upper secondary school.

This is more likely in mathematics than in some other subjects. Until recently, education for teachers in grunnskole in Norway has included little mathematics, and little associated didactics of mathematics. The situation has improved over recent years, so that teachers at all levels have basic courses in mathematics and didactics and there is possibility for teachers to choose to study mathematics and didactics beyond the basics. However, there are many teachers still within the grunnskole system who have little formal competence within the subject and possibly little understanding of mathematics didactics.

There are many factors which can contribute to pupils' underachievement in mathematics, but it seems clear that teachers' confidence and competence in mathematics and in ways to approach the teaching of mathematics is one of the most important.

The proposed consortium

Didacticians at 5 colleges in Norway (in Bergen, Bodo, Kristiansand, Oslo and Trondheim) agreed to work together towards the strengthening of mathematics education in Norway. Each group has its own proposed project and its own goals, but all are based in the development of learning and teaching mathematics, and all are based in a philosophy of inquiry communities. During work in

the individual projects the consortium will meet regularly, share outcomes and issues in their projects and address questions fundamental to improving mathematics learning and teaching widely.

In the beginning a main task will be to develop a set of research questions that can be addressed in all projects. Starting points involve ways in which the individual projects enable development in learning and teaching mathematics and in mathematics teacher education. For example, how are practicing teachers involved in each of the projects? What opportunities for learning do the projects afford the teachers? How can pupils' learning of mathematics be seen to develop? In what ways do the projects contribute to the education of teachers? What can we learn more broadly, from issues and outcomes reported, about patterns of developmental activity that promote better teaching and learning of mathematics? Indeed what is meant by "better" learning and teaching?

In accepting our proposal, NFR emphasized the importance of the Consortium. Thus, funding should cover expenses for consortium meetings and conferences, both between the principal researchers in the projects and more widely for participants in and beyond the projects. Reporting from the consortium will be a synthesizing of knowledge addressing mathematics learning and teaching beyond individual projects. How this is to be achieved will be part of our discussion in meetings from the beginning.

Teaching better mathematics.

Teaching better mathematics is a two-edged title. It proposes that *teaching* will be *better*, and that *better mathematics* will be taught. We start with the second of these, *mathematics*. Mathematics is known as the language of science, an ideal tool for modelling scientific theories, deriving qualitative consequences from them and forecasting events. We see its uses in areas as diverse as space research, weather forecasting, geological exploration, and high finance. However, mathematics is also a language for everyday life, a central part of human communication, and a means of articulating patterns, relationships, rationality and aesthetics. For all these reasons, mathematics is a central subject in the school curriculum for students at all levels and lends itself as a tool and way of thinking to many other subjects. Pupils should learn not only to use and apply mathematics rules, processes and formulae but also develop principled understandings of mathematics, ways of thinking mathematically and ways of tackling a wide range of problems using mathematics. It is this that we mean by *better mathematics*. *Better teaching* of mathematics follows: it is teaching that offers pupils opportunities for a rich experience of mathematics through which principled understandings can develop and higher levels of achievement result. Teachers must know mathematics deeply and in principled ways themselves so that they can support and challenge pupils appropriately at all levels. The education of teachers must achieve such deep and principled knowledge as well as knowledge in didactics and pedagogy of mathematics that enables the quality of teaching we espouse. Here we see also challenges for didacticians with regard to working for such goals of teacher and teaching development.

The projects start from these ambitious goals. They involve a research and development programme such that all participants are researchers and that engagement in research is a developmental tool as well as creating scientific knowledge. We explain these ideas further below. It is fundamental that teachers define their goals and boundaries within the projects and that didacticians respond to teachers declared goals offering knowledge and expertise according to negotiated agreements.

Inquiry and Community

Our philosophy of learning and teaching in the consortium is based within two fundamental theoretical principles, those of *inquiry* and *community*, and we base all our work in development and research on the building of *inquiry communities* involving, "a willingness to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to make answers to them" (Wells, 1999).

To inquire means to ask a question; to make an investigation; to acquire information; to search for knowledge: Thus we see inquiry to be about questioning, exploring, investigating, and researching.

In these ways it is central to approaches and heuristics in mathematical problem solving (Polya, 1945, Mason, Burton and Stacey, 1981, Schoenfeld, 1985). It is also central to practitioner research designed to develop teaching (Elliot, 1991; Cochran Smith and Lytle, 1999). It leads to metacognitive awareness and ability to take responsibility for learning (Wells, 1999; Mason 2001). The essence of inquiry as a basis for effective learning is a position of looking critically, in a positive way, at whatever we are engaged in, asking critical questions that take us deeper into the substance of our activity. We aim to become more knowledgeable about our practice and more able to function wisely and effectively. This applies to doing and learning mathematics, to developing approaches to teaching, and to conducting research.

We recognise that we are humans working together to achieve mutual goals, but that we have different focuses and histories. Social groupings such as school, college, family and friends; social structures such as society and schooling; political systems; economic factors; cultural factors; all influence our working lives. We engage in certain forms of practice and these practices are what we seek to understand more clearly in order to develop and improve what we do. We see ourselves as forming a *community of practice*, a social grouping affording interactive engagement and an environment for learning in an area of mutual action, interest or concern (practice) (Lave & Wenger, 1991; Wenger, 1998). However, we go further in developing a community of inquiry in that we seek to look critically at the practices in which we engage and develop these to achieve better outcomes. As a community, we look critically at our practices while engaging in them in order to become more knowledgeable and afford opportunity to improve.

How this builds on the KUL Projects

Researchers in mathematics education at Agder University College (AUC) have been involved in two projects, closely related to TBM, in the NFR KUL programme (Kunnskap, Utdanning og Læring – Knowledge, Education and Learning). These are *Learning Communities in Mathematics* (LCM) and *Information and Communications Technology in Mathematics Learning* (ICTML). Both of these projects are founded in a philosophy of developing inquiry communities for improving teaching and learning in mathematics. Examples of communities include the teachers in any school, or the group of didacticians in mathematics education at the college. Researcher in both projects have worked hard to develop inquiry communities with the aims and goals as set out briefly above.

In the KUL-LCM project a central questions has been what is needed for teaching to develop so that pupils can engage with mathematics in ways that enable their understanding and enjoyment. In the ICTML project, the central question is what ways teachers can use ICT to provide pupils with effective learning experiences in mathematics.

These projects have completed two phases of joint activity between teachers and didacticians and are in the middle of Phase 3 currently. Phase 1 was a phase of community building, in which ideas of inquiry were introduced and in which joint exploration took place in workshops at the college and in schools. 8 schools have participated, with commitment from the school leaders and at least three teachers from each school in the projects. In Phase 2 work has focused on what inquiry looks like in schools, both in planning for teaching and in the classroom. We held a major conference in September 2006 which promoted the ideas of and learning from the projects through presentations of activity and findings. Various other conference presentations and publications have resulted so far and others will follow as the projects move into a concluding phase. A list of publications for the KUL projects can be found at *websites*: <http://fag.hia.no/lcm/> and <http://fag.hia.no/iktml>

In addition, a longitudinal study of pupils' achievement in KUL project schools from grade 1 to grade 13 has been conducted. This is an extremely important source of data from which analyses are already producing results (see publications list). Ongoing data collection allows us to trace achievement and development in achievement long term. We therefore hope to continue this study as part of the TBM project.

The communities that have developed in the KUL projects are in progress of changing teaching and learning cultures in mathematics in project schools. An important learning outcome concerns the nature of such changes in culture and the issues raised and faced in the processes involved. Schools, teachers and didacticians now know a great deal more about the nature of inquiry communities, modes of implementation and issues in their growth than at the beginning of the KUL programme. It is only over time that development takes place and real learning occurs. What has been put in place and learned over these projects so far stands as building blocks for the future. The TBM projects builds on what has been learned in KUL.

A description of each of the individual projects at the 5 colleges can be found at <http://fag.hia.no/tbm>.

Outcomes, expectations and implications from the TBM projects

Evaluation of ongoing activity

Evaluation of practices will be ongoing with developmental activity. Each project will monitor its own activity and respond contingently. The consortium will act as a monitoring organ through discussion and critical questioning of each of the projects at periodic consortium meetings. We will plan an external evaluation of each project in time to influence project completion.

Sustainability

A most important consideration will be how development can be sustained after the life of the projects. The nature of these projects is such that all people concerned are responsible for design and development and their practices are the focus of design and development. There is an expectation that development is fundamental to practice and should grow to be a basic element of practice. Thus sustainability will be an ongoing consideration in activity and research into activity. The issues that arise in interpreting theoretical aims for the project in realistic practical terms will be tackled as the projects develop, and not left to 'solve at the end'. These issues, and the ways in which they are addressed, will be an important part of what is learned in the project, and will make an important contribution to eventual reporting from the project. Thus sustainability will be an integral part of research and development.

New practice, new knowledge, new theory and their communication

In developmental research, new practices emerge continually through the process of design, activity, reflection and feedback. It will be central to our work in the projects to document carefully both activity and reflections on activity in order to chart progression. Close relationships developed in the communities will enable a critical inspection of what is being learned, and research findings will support a critical synthesis. It will be the responsibility of the didacticians to formalize the processes involved to extract indications that form the basis of new knowledge and theory.

In all of the projects, all participants will engage in research writing. This will include records from teachers' planning and reflection in schools, pupils' work, student teachers' portfolios, analytical memos, research reports. All writing will contribute to publications to inform and inspire practice and to communicate research findings. These will take a wide variety of forms from local reports, through short articles in professional journals to more scientific articles in research journals. Teachers and didacticians will make presentations at conferences locally, nationally and internationally. Research publications will communicate such findings in the scientific community. A book or books will be written to capture the developmental processes, issues in establishing inquiry communities, outcomes for teacher and pupil learning, and knowledge about the improvement of mathematical learning in schools.

Dissemination

Disseminating what is learned in these projects goes beyond the publication of reports and articles. What is learned by participants in these projects will be hands-on knowledge about teaching and learning that can be described but not handed on through publications. Although the written

synthesis will be a valuable communication tool, it is a serious question how the hands-on knowledge can be communicated. One of the issues we have to tackle as part of these projects is how others can learn from our activity and thinking as it is happening, rather than by reading about it afterwards. In a very real way, what is learned is learned through participation, and to learn one has to participate. We will seek support from the Research Council of Norway and Utdanningsdirektoratet /Kunnskapsdepartementet to address this conundrum.

Project personnel development and funding

In all the projects, participants in the colleges are didacticians and teacher educators at a variety of levels. For all, participation in the projects will contribute to their development and to capacity building locally and nationally. Although the colleges will support much of this development of their staff, some project funding is also sought for development of høgskolelektorer to førsteamanuansis or førstelektor positions, and enabling some to study for PhD degrees. Such development is an important outcome of the projects. The community at HiA with its doctoral programme and research experience will contribute to developments in the other colleges. The consortium will be an important means for such development to take place.

Project Management

The project director will set up a policy group for managerial decisions. Project activity will be subdivided according to particular forms of activity, with individual leaders in charge of subdivisions. A project coordinator/secretary will assist the policy group as appropriate. Meetings of the whole group of didacticians will be frequent to ensure development of common understandings in activity across schools. A steering committee with suitable representation will be convened.

Synthesis of knowledge within the consortium and development of mathematics education in Norway

Consortium activity will consist mainly of communicating issues from practice in the five projects and looking critically at what we are doing and achieving. We will compare issues and findings across the projects for common threads and counter indications, to enable a more general picture of issues in mathematics learning and teaching to emerge, particularly in relation to the new curriculum and pupils' achievement in mathematics. Records from consortium meetings will be an important source of data for analysis in respect of the overall learning across the projects. We will open up the consortium debates to others in the field in and beyond the five colleges to aid the dissemination process. Such opportunities, together with presentations at national events will contribute to the development of the mathematics education community in Norway and prepare the ground for a national culture in which achievement in mathematics is raised generally.

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