Teaching better mathematics

Learning of mathematics through activities and communication

Main aims and goals of the project

This research project incorporates two schools, staff from the mathematics department as well as students on various levels from Sør-Trøndelag University College (abbreviated STUC). The main goals of the project are

- to improve the learning of mathematics in schools by fostering relational understanding
- to raise the competence in mathematics teaching for teachers in school
- to improve teacher education, both at bachelor and masters level
- to raise the research competence of staff members

To achieve the goals the people involved in the project will work closely together. This work will take place in various settings, involving didacticians, teachers, student teachers and pupils – the various groups of people working together in various combinations. In our work we build on a model for *communities of inquiry*, which is elaborated on in the overall description of TBM http://fag.hia.no/tbm/.

The project aims at developing new knowledge about how children learn mathematics and is based on a desire that teaching of mathematics should lead to relational understanding (Skemp, 1976) of key concepts. Some central issues to be looked at are:

- Learning of mathematics in a variety of environments – outdoors, in the local community
- Development of mathematical language, from informal “pre-school language”
- Transition between the various levels (barneskole-ungdomsskole)
- Challenges for the subject of mathematics following from implementing the new national curriculum (Kunnskapsløftet)

The project will be carried out in close contact with the schools involved.

Background to the project

Mathematics education in Norway

The project has its background partly in the situation for mathematics education in Norway in general, as described in http://fag.hia.no/tbm/, and partly in the situation for mathematics education at STUC.

At STUC we have systematically worked on improving the level of mathematics in the initial teacher education, surpassing the requirements that have been decided on nationally. Since 1992 we have had a separate study programme emphasising mathematics and science (allmennlærerutdanning med vekt på realfag). This programme was reorganised into a project in 2001 where several new principles were incorporated. One of the new principles was to strengthen the links between theory and practice. Over the last few years we see an increasing interest among students to apply for this programme.

From 2002 we have been part of a joint project developing courses for in-service training in mathematics. This project was initiated by SOFF (now NUV), and STUC, together with Oslo University College, was in particular responsible for developing two courses (each 30 credit points) focussing on mathematics for teachers in primary school (barnetrinnet). This project is finalised in November 2006.

As of 2005 we have established a master's programme in mathematics education, and the first group of students was admitted to this programme in August 2005. A new group (nine students) was admitted in August 2006. (See http://www.alt.hist.no/adm/studier/opptak/master/mastermatte05.php)

The current project

The current project is closely linked to the implementation of the new national curriculum, Kunnskapsløftet (UFD, 2005). This curriculum does not specify particularly favoured methods for working with mathematics in schools. On the contrary, it is a basic principle that each teacher or group of teachers should have freedom in choosing their own ways of working. The curriculum only specifies which goals within mathematics that the pupils are supposed to achieve on various levels. Because of this freedom one may expect a variety of approaches in various classrooms and it will be of interest to study, and maybe compare, the effect of implementing certain ways of working.

The five basic skills are fundamental in Kunnskapsløftet. Emphasis on the ability of expressing oneself orally and in writing can be seen as novel approaches to mathematics compared to what traditionally may have been regarded as important in the subject. It will be of interest to develop more knowledge about what the five basic skills really mean in mathematics. In this project we will be concerned with the development of mathematical language and concepts, and how reading, writing and discussing will enter into this development process.

Activity and communication are central issues in the project. Since the implementation of L97 (KUF, 1996) there has been a considerable focus on student activity in mathematics. This focus has been natural due to the constructivistic principles on which L97 is founded. The evaluations of L97, both in general (Haug, 2003) and more specifically in mathematics (Alseth, Breiteig, & Brekke, 2003) have shown that despite the high level of activity the learning outcome may not be satisfactory. International studies, (e.g. Säljö, Riesbeck, & Wyndhamn, 2001) have also documented that activities alone will not generate learning. Recently, one can observe a trend that the value of activities is seriously questioned, and arguments are raised in favour of the view that activities should be replaced by drill and practice (Gradowski & Sigmundsson, 2006). However, activity approaches were introduced due to dissatisfaction with the outcomes of drill and practice (Jaworski, 1994). On a fundamental level this has to do with views of knowledge and learning. It is important to challenge the perception of knowledge as something that can be transferred, as opposed to constructed (von Glasersfeld, 1995) or developed through social participation (Lerman, 2000). Thus, we believe that it is important to try to understand and characterise the learning that takes place through various ways of working. Such knowledge will help the teacher to choose ways of working suitable for the specific learning that he/she wants to achieve in various situations. Making a distinction between conceptual and procedural knowledge (Hiebert & Lefevre, 1986), and discuss how these two kinds of knowledge could be achieved would also seem relevant.

It is important to realise that the endeavour to change classroom practice takes time. On a fundamental level this has to do with views of knowledge and learning. It is important to challenge the perception of knowledge as something that can be transferred, as opposed to constructed (von Glasersfeld, 1995).

The work in the project would fit well into the developmental cycle with design/innovation/reflection/feedback as described in (Jaworski, in press).

Relation to ongoing and previous work

The project builds on, and continues, work already going on at Saksvik school in Malvik, outside of Trondheim. A central theme in the project at Saksvik is to investigate the potential of student activities for inducing learning of mathematics. In particular, the project points to the role of outdoor activities, and to study the *kind of learning* that takes place in an outdoor environment, and to what extent this learning is different from the learning in the traditional classroom.

As a second school in the project we have Hommelvik ungdomsskole. This school wants to develop new and more varied ways of working in mathematics, with a focus on student active ways of working. Representatives from the school have expressed a desire to change practice in mathematics. We will assist in formulating more precise goals for their work. It seems as there may be some overlap with the aforementioned Saksvik project in the way they think. In total the two schools will represent all levels in the span from grades 1-10. Both schools are situated in Malvik kommune, and they are part of a developmental project with funding from Utdanningsdirektoratet's Program for skoleutvikling (http://www.utdanningsdirektoratet.no/templates/udir/TM_Artikkel.aspx?id=1437).

From 2001-2004 STUC was involved in a development project in mathematics at Vikhammeråsen grendaskole. Here we worked closely with the teachers to develop student active ways of working with mathematics without being dependent on one dedicated text book. In the proposed project we will build on experiences from this project (Rønning & Troøyen, 2004).

Theoretical background

Teaching and learning of mathematics

The ultimate goal of the project is to contribute to the development of knowledge about how children learn mathematics, more specifically how to foster relational understanding of key concepts. We draw on various theories of learning, but given the foundation of the project in the concept of communities of inquiry (Jaworski, 2004), we will in particular draw on sociocultural and constructivistic theories of learning (Steffe & Gale, 1995). The concept of inquiry means that the various actors in the learning and teaching situation enter with an open mind, eager to develop knowledge together. In this context the concept of negotiation of meaning will be relevant. Here theories about dialogic learning (Alrø & Skovsmose, 2002) and interactionistic theories (Voigt, 1995) will form a relevant framework for analysis.

Even though we acknowledge and value the importance of the competence and experience that each of the participants bring into the inquiry community we are aware of the different roles played by teacher and pupil, where the teacher has the role of the more competent person. When working with activity based learning we are in particular interested in exploring the role of the teacher when it comes to development of knowledge and what kind of knowledge that pupils will develop when engaging in activities.

Research domains and questions

Some broad research topics could be

- Development of mathematical language. Students' own development of concepts and teacher support.
- Mathematical knowledge developed in various environments and with various ways of working
- Mathematics in interdisciplinary work
- Building on existing knowledge in the transition between levels
- Student teachers' development of mathematical and didactical knowledge in the practice field

The project can be seen to focus on a diversity of mathematical topics. Problem solving as a way of working will be central (Polya, 1957; Schoenfeld, 1985). Based on the expressed interests from the participating schools and the interests of the didacticians in the project, the project can be seen to be separated into two, partly overlapping, topics.

1. Learning through activities.

Research questions to be addressed here:

- Identify and characterise the learning that takes place when pupils engage in a certain activity.
- Identify the need for, and the effect of, teacher intervention when pupils engage in a certain activity
- Relate the observed learning to the learning goals in Kunnskapsløftet

2. Development of mathematical language

Research questions to be addressed here:

- Identify the qualities and restrictions of informal mathematical language
- How can informal mathematical language form a basis for fostering relational understanding?
- How can various activities and learning environments act as catalysers for developing the mathematical conversation?

In topic 2 a link will be made to researchers working in the school subject Norwegian to study the pupils' writing processes.

Research methodology and research training

Part of the project design is that all participants will be researchers in the project. Therefore it will be necessary to organise research training for the teachers involved. This will be the responsibility of the didacticians. It may be feasible to combine research training for teachers with parts of the methodology course that is provided for the masters students.

Didacticians will collect data as far as possible from all activity for formal analysis. This will mainly involve qualitative methods. Such methods of data collection will include observation in classrooms, with audio and video recordings of classroom activity; audio-recorded interviews with pupils and teachers; written reflections on experienced activity; written documents from lesson planning, classroom materials and pupils' work. Analysis of such

data will use a variety of techniques including those of data reduction, categorization, and fine tuned analysis of discourse. Qualitative software will be used to assist analyses.

Given the relatively small scale of the project it may seem that quantitative data are less appropriate, but we will seek possibilities for scaling up the amount of data by looking at common interests between the collaborating schools and also between the partners in the consortium.

The data collected for formal analysis will complement that of teachers in the design cycle. Both forms of data will be used as part of the developmental process. For example, video of lessons, as well as forming a basis for formal analysis, will be used as a basis for discussing classroom processes amongst groups of teachers and didacticians to strengthen knowledge of teaching and a deeper awareness of classroom actions.

Teacher education

How the project links to teacher education

From 2005 we have started a master's programme in mathematics education (http://www.alt.hist.no/adm/studier/opptak/master/mastermatte05.php). The aim of this programme is to educate teachers with a strong background in mathematics and didactics of mathematics emphasising a principle of developing knowledge through linking theory with practice.

The students enrolled in this master's programme will enter the proposed project in a natural way. The research areas of the project fit very well with central topics in the plan for the master's programme. Both in their first year and for their masters thesis the students will need access to schools in order to carry out their research. With supervisors as researchers in the project, this would be a good model for carrying out the work for the masters thesis. In the academic year 2006-2007 four of the students in the second year of the master's programme are doing their thesis in connection with the project, and all of the nine first year master's students are doing their field work in the two schools linked to the project.

For students in first and second year of pre-service education we have developed a model with field studies. These are small projects to be carried out in the classroom, resulting in classroom observations to be analyzed, linked to theory. The group of didacticians involved in the proposed project will teach both in the master's programme and in pre-service education, making it easy to link students from both programmes to the project.

The project will also provide various opportunities for in-service training for the teachers involved. Didacticians and teachers from the collaborating schools will work together in various ways, and in smaller and larger groups, depending on the situation. This will in itself act as an in-service training for the teachers involved. A workshop for the teachers at both schools will be organised in November 2006. In addition there is the possibility of offering courses that give credit points to the teachers. We have developed various courses suitable for this purpose, based on a seminar/distance education model that could run along with the project. We have courses available that are suited for teachers with very little formal background in mathematics, as well as courses for those that have a stronger background.

How the project will strengthen teacher education

A number of public documents have addressed the need for strengthening the professional relevance of pre-service teacher education and a closer connection between theory and practice. Work in this direction has been going on at STUC for several years, notably initiated through 'Realfagsprosjektet' (Måsøval, 2003; Måsøval, Rønning, & Smidt, 2003) starting in

2001. One of the pillars of this project was to enhance the use of the practice field as a learning arena for student teachers. This led to the development of field studies – designated tasks for the students to be carried out at the placement schools, outside of the regular practice periods, focusing on a special topic and with clearly defined objects for investigation and learning. In some instances this has led to small research projects involving student teachers and didacticians (Rønning, 2004a; Rønning, 2004b).

The concept of field studies has been incorporated into the pre-service teacher education programme on a permanent basis after the ending of the project period. The idea has been further developed into the master's programme – in particular in the course Learning and Teaching of Mathematics in which the students are assigned a number of tasks to be performed with children, and subsequently reported on in the masters class. One of these tasks is developed into an essay which is a part of the final examination in the course.

An important aspect of this work is to develop the identity of mathematics as a subject in teacher education, and to identify differences between this and mathematics as a school subject or as a science.

Through the proposed project the link between theory and practice will be strengthened further, both on the basic and the more advanced (masters) level of teacher education.

Relationships with schools

The project is based upon each school having its own rationale for entering the project. It is vital that the school is committed to doing developmental work in mathematics. It will involve the whole school community and must be supported by the head teacher. It is also important that each teacher is committed to the project. Each school will formulate its own list of questions to be looked at. This will be done in collaboration with us and such that it fits in the inquiry model.

The activity of the project

Initial activity in schools

The schools will work on their own project and formulate a plan which will be embedded into a proposal submitted by the school owner. Didacticians will assist the schools in refining this plan.

Initial activity in the college

Didacticians working together have formed a group discussing areas of common interest to the whole project. The group consists of six persons, and it is split in three pairs – one pair working towards Hommelvik ungdomsskole and two pairs working towards Saksvik skole. Here one pair is working with the lower grades and one pair with the upper grades

Mathematics workshops

An important part of the project, involving all teachers and didacticians, and to some extent students, is the mathematics workshops. Here we will get together all the teachers in the participating schools to work with didacticians in an inquiry based atmosphere. Topics to be addressed in these workshops will be determined in dialogue with the participants, but the aim of the workshops is to raise the level of competence among the teachers both in mathematical topics and in topics from didactics of mathematics. Whatever topic we will work with it is a basic principle that the activities will be marked by a problem solving and a dialogic approach.

Staff development

In the description of the Program for praksisrettet FoU staff development and recruiting researchers are emphasised as important parts of the programme. It appears to be a national challenge to develop people with a research competence in didactics of mathematics. Experience from work in committees evaluating applicants for positions both at university colleges and universities, indicates that the availability of people with such competence is scarce. In our department we also see the need for developing the level of competence. The project aims at staff development through the model where we form teams consisting of people with a varying degree of research experience. Participating in the consortium further strengthens the possibilities of competence development. Linking a stipendiat to the project would also be a step in this direction. A stipendiat position, funded by the project, has been announced in October 2006. The person in this position will do his/her research at STUC, but will follow a PhD programme at Agder University College.

When STUC proposed to launch a master's programme in mathematics education, the scientific committee evaluating, and recommending, the proposal commented that there was a need for us to develop our scientific competence in the field, and to increase the research activity. We have started this process, and we can point to several steps that have been taken in this direction. However, the process needs to go on and the proposed project is a part of the necessary development to build a strong platform for offering the master's programme.

Application of the results

The project is both a research project and a development project. Through the project the schools will try out a number of different teaching strategies and designs. These will be discussed and analyzed in groups consisting of teachers and didacticians. Through these discussions and analysis we will identify stronger and weaker points in the various strategies and designs. This will lead to a better understanding of what will result in good learning, and why.

Dissemination

Presentations at conferences

Conferences will be organised internally in the project and in the consortium. This will be the first level where results from the project will be presented. Furthermore, we will aim at presentations in various types of conferences. The conference FoU i praksis, organised annually in Trondheim, will be one natural arena. Other natural arenas, more focussed on mathematics, will be the NORMA conference (next conference in 2008) and the PME conference, organised annually. Another arena, aiming more at the practising teacher, would be the annual summer course arranged by LAMIS.

Research articles

Didacticians, possibly with teachers or students as co-authors, will aim at publishing papers in scientific journals and in proceedings from conferences such as NORMA and PME.

Articles in professional journals

The nearness to the practice field, which is a cornerstone in the project, implies that results from the project will be suitable for presentation also in journals that are not characterised as research journals. One natural example in this group would be the journal for mathematics teachers, Tangenten.

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